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Self-Optimizing Water-Filling Power Allocation: A Hybrid Fractional Frequency Reuse Way

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Outline

Background and Motivation

- Due to the development of 5G networks, wireless data traffic is accelerating at an unprecedented rate. **To increase a cellular network's spectral efficiency (SE), the fundamental approach is handling the power allocation (PA).**
- Among all these heterogeneous cellular networks, the basic structure is the hexagonal grid for the BS. **And by applying frequency reuse (FR) methods, the utilisation of spectrum resources will be more efficient and serve a larger area with flexible FR methods.**
- Fractional Frequency Reuse (FFR) and Soft Frequency Reuse (SFR)[1] schemes have been evaluated as two **inter-cell interference (ICI) mitigation methods** to increase network SE in a Orthogonal Frequency Division Multiple Access (OFDMA) based multi-cell deployed next generation wireless network[2].

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[1] (Abbas et al. 2020; Novlan et al. 2011)

[2] (Han et al. 2017; Garcia-Morales, Femenias, and Riera-Palou 2019)

Contributions

The major contributions are summarized as follows:

- We analysis the overall spectral efficiency of the multi-cell network applying different power allocation algorithm in **two different frequency reuse scenarios given the perfect CSI**.
- We develop two hybrid power allocation algorithms, the **SOWF** is to reach the optimal SE with some sacrifice in complexity, and the **FLWF** is to reduce the complexity in computing the network SE.
- We also **evaluate the complexity of the proposed algorithms** compared to tranditional algorithms.
- The network SE in sFFR and SFR of different algorithms are compared based on the generated analytical equations and then confirmed using **Monte-Carlo simulations**.

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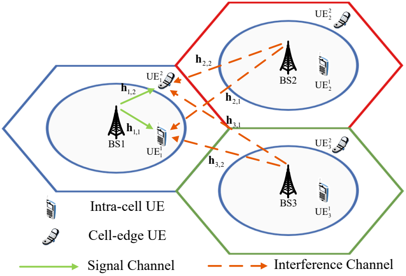
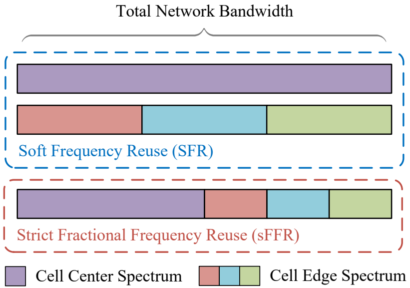
System Model

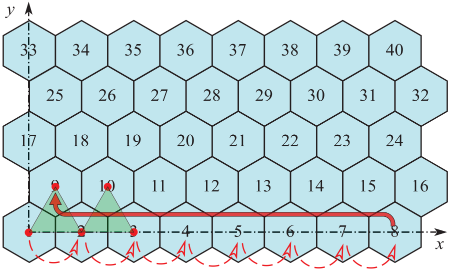


Frequency Reuse Schemes



- Apply sFFR: No co-channel interference between the same cell;
- Apply SFR: Severe interference but save spectrum resources;
- Divide the users into two group, one in the cell center and another in the cell edge, and apply different power allocation algorithms.

Here is a method of multi-cell generation

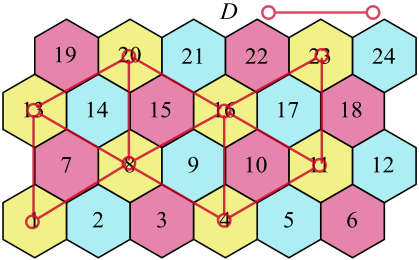


- A network consists M cells $M = R(row) \times C(column)$;
- From left to right, from bottom to the top;
- Use the mathematical relation between adjacent cell centres.

The green triangle shows the relation between each cell centre and also help to find the relation between the odd and even rows.

BFS for frequency allocation

In this paper, we use the Breath First Search to determine the cell groups that share the same frequency channel:



- ❑ Define a frequency reuse distance D for IFR scenario;
- ❑ Begin at cell 1, one group at a time;
- ❑ Different color represcents different frequency reuse group.

This search method help reduce the complex process of constructing a matrix about the frequency reuse group of cells.

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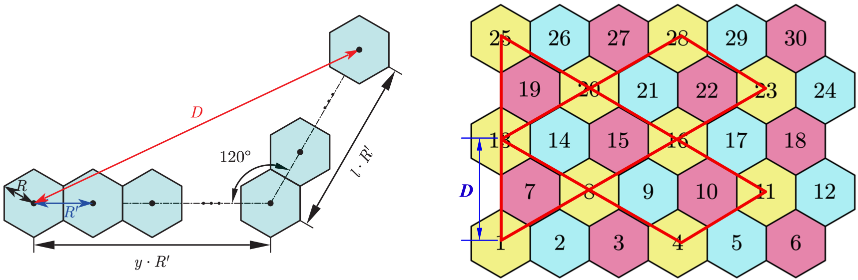
The schematic of identifying IFR cell groups



- The Frequency Reuse Factor (FRF), $N = \frac{D^2}{3R^2}$;
- Based on cosine law for sides, $D = \sqrt{3N}R$;
- Apply **Breath First Search** algorithm to get corresponding group information.

The capacity analysis of IFR with different FRF

Consider a M-cell network, the capacity for the frequency channel f in the cell i can be calculated by

$$C_f^{i,u} = \sum_{f=1}^{N_i} \log_2 \left(1 + \frac{|h_{i,i}|^2 p_i^u}{\sum_{j=1}^{M} \rho_{j,i} |h_{j,i}|^2 p_j + \sigma^2} \right) (i \neq j). \tag{1}$$

Thus, the capacity of a network using IFR can be derived as

$$C_{\text{IFR}} = \sum_{i=1}^{M} \sum_{u=1}^{U_i} C_f^{i,u}. \tag{2}$$

$h_{i,i} \rightarrow$ channel coefficient, $\rho_{j,i} = D_{j,i}^{-\alpha}$ ($D_{j,i} \rightarrow$ distance between cell j and i)

Strict Fractional Frequency Reuse (sFFR)

The network capacity analysis of sFFR

The capacity of all cell centres in the network is given by

$$C_{\text{centre}} = \sum_{i=1}^{M} \sum_{u=1}^{U_i^1} \sum_{f=1}^{N_i} \log_2 \left(1 + \frac{|h_{i,i}|^2 p_1^f}{\sum_{j=1}^{M} \rho_1^{j,i} |h_{j,i}|^2 p_1^f + \sigma^2} \right). \tag{3}$$

The capacity of all cell edges in the network is given by

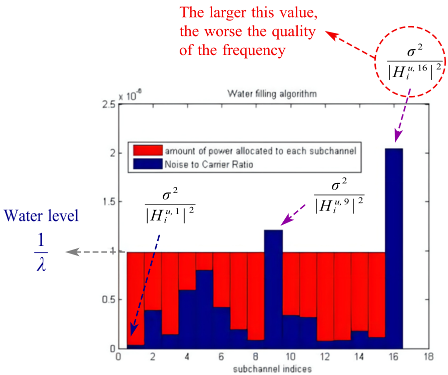
$$C_{\text{edge}} = \sum_{i=1}^{M} \sum_{u=1}^{U_i^2} \sum_{f=1}^{N_i} \log_2 \left(1 + \frac{|h_{i,i}|^2 p_2^{u,f}}{\sum_{j=1}^{M} \rho_2^{j,i} |h_{j,i}|^2 p_2^{u,f} + \sigma^2} \right). \tag{4}$$

Both IFR and FFR are still rigid and inflexible

- Some cells may not have enough frequency channels to serve their users even if some other cells do not need their frequencies;
- Even if there are abundant frequency resources, they cannot be used to improve the capacity for the active users.

The network capacity analysis of SFR

$$C_{\mathsf{SFR}} = \sum_{i=1}^{M} \sum_{u=1}^{U_i} \sum_{f=1}^{N_i} \log_2 \left(1 + \frac{|h_{i,i}|^2 p_i^{u,f}}{I_1 + I_2 + \sigma^2} \right), \\ I_1 = \sum_{j=1}^{M} \rho_1^{j,i} |h_{j,i}|^2 p_1, I_2 = \sum_{j=1}^{M} \rho_2^{j,i} |h_{j,i}|^2 p_2^{u,f}, \tag{5}$$



Start with cell 1

Start with user 1

$$\text{Repeat}\begin{cases} p_i^{u,f} = \left(\dfrac{1}{\lambda} - \dfrac{\sigma^2}{|H_i^{u,f}|^2}\right)^+, \\ \sum\limits_{f=1}^{N_i} \left(\dfrac{1}{\lambda} - \dfrac{\sigma^2}{|H_i^{u,f}|^2}\right)^+ = P_{\max}, \\ \text{Move to another user} \end{cases}$$

Until convergence

Get the optimal power allocation scheme

- The total capacity with constraining for one user is given by
- $\max\limits_{\{p_i^{u,f}\}} C_{\mathsf{SWF}}^u = \sum\limits_{f=1}^{N_i} \log_2\left(1 + \frac{p_i^{u,f}|H_i^{u,f}|^2}{\sigma^2}\right)$, s.t. $\sum\limits_{f=1}^{N_i} p_i^{u,f} \leq P_{\max}.$

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Forward-Looking Water-Filling (FLWF)

Forward-looking Water-filling constructs a OFDMA cognitive radio network that approaches forward-looking equilibrium(FE) (Ren and Wong 2018)
The power allocation for user u at time t is updated by (6) using the previous power allocation information,

$$\begin{aligned} p_u^t[f] &= \left(w_u^t - \frac{\left(c_u^t[f]\right)^2 + \varphi_u^t[f]\left(p_u^{t-1}[f]\right)^2}{c_u^t[f] - \varphi_u^t[f] p_u^{t-1}[f]} \right)^+, \\ \varphi_u^t[f] &= -\sqrt{\frac{c_u^t[f]}{2c_u^t[f] + p_u^{t-1}[f]}} \quad \forall u. \end{aligned} \tag{6}$$

where $c_u[f] \triangleq \sigma_u[f] + I_u[f]$ corresponds to the overall noise on the frequency channel f for user u.

Self-Optimizing Water-Filling (SOWF)

When it comes to SFR scenarios, the classical SWF can hardly satisfy many users for its utilizing some intermediate frequency channel and introduce a strong co-channel interference. Self-optimizing parameter ξ that helps manage the level of interference.The power updating scheme can be derived as

$$
\begin{aligned}
p_i^{u,f} &= \begin{cases} \left(w_u^t - \gamma_i^{u,f}\right)^+ & \text{if } \gamma_i^{u,f} \leq \xi_i^{u,f} \\ 0 & \text{if } \gamma_i^{u,f} > \xi_i^{u,f} \end{cases}, \\
\sum_{f=1}^{N_i} \left(w_u^t - \gamma_i^{u,f}\right)^+ &= P_{\max},
\end{aligned} \tag{7}
$$

where $\gamma_i^{u,f} = \frac{\sigma^2}{|H_i^{u,f}|^2}$, and the self-optimizing $\xi_i^{u,f}$ can be calculated by

$$
\tilde{\xi}_i^{u,f} = \max\left(\xi_i^{u,f}, \frac{\sum_{u=1}^{U_i} c_u^f \operatorname{sgn}\left(p_i^{u,f}\right)}{\sum_{u=1}^{U_i} \operatorname{sgn}\left(p_i^{u,f}\right)}\right). \tag{8}
$$

Hybrid Power Allocation Algorithm

To serve more user and reduce the co-channel interference. In this paper, we design a hybrid FFR power allocation iterative algorithm, as shown in Algorithm 1.

Algorithm 1 Iterative-Based FLWF/SOWF PA algorithm

1: Initialize parameters M, $P_{\max}$, N, U, $p_{\max}$, ρ, h, σ^2, w
2: Set the maximum iteration times $T_{\max}$ and the convergence accuracy δ, set the initial iteration index $t = 0$.
3: **while** $\sum_{u=1}^{U_i} \sum_{k=1}^{N_i} \left| p_u^{t+1}[k] - p_u^t[k] \right| \geq \delta$ and $t \leq T_{\max}$ **do**
4: Calculate $p_u^t[k]$ using (6) or (7) for all the cell center.
5: Calculate the SE for all the cell cente.
6: Calculate the SE for all the cell edge.
7: $t = t + 1$.
8: **end while**
9: Calculate the overall network SE: $S_{\text{FFR}} = S_{\text{center}} + S_{\text{edge}}$.

- First, find the optimal values of all variables of (6) or (7) in each iteration.
- After that, the network SE can be calculated based on the power allocation scheme.
- And to find the optimal SE, we get into the next iteration with former information until it converges.

Outline

Simulations

Parameter Settings

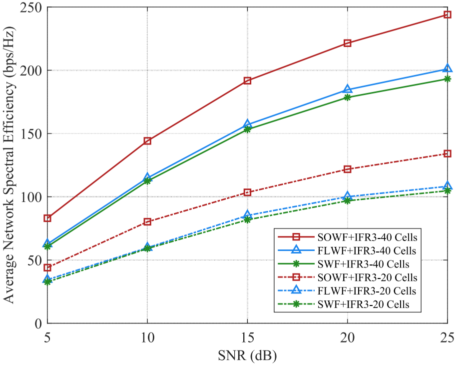
- A 30-cell network
- Six frequency channels for a cell centre (edge)
- Radius of the cell centre (edge): 150 m (200 m)(Sun et al. 2018)[1]
- $U_c = 12$, $\alpha = 3$, $T_{\max} = 300$,$\delta = 10^{-3}$
- With 12 UEs: $p_{\max} = 35$ dBm, $P_{\max} = 46$ dBm

Algorithm Comparisons
Different cell number and frequency reuse schemes

- SWF/IFR1+ IFR3
- FLWF + IFR3
- SOWF + IFR3

[1]This cell radius setting has weaker interference but has lower spectral efficiency in most cases.

SE comparison of different schemes

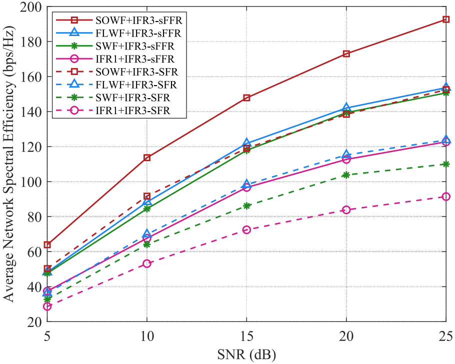


- ✓ SOWF + IFR3 has the **highest** SE for both the 20 and 40 cell configurations;
- ✓ FLWF + IFR3 reduce the computational complexity and reach a slightly **3%** improvement of SE ;

Scenario \ Algorithm	SWF	FLWF	SOWF
sFFR	105.249	59.422	140.353
SFR	150.672	92.342	207.426

Besides, for the sFFR scenario, the FLWF affords savings in numerically computing the optimal power allocation for each user as the number of terms in the power updating drops from 105.249 to 59.422 in sFFR scenario.

SE comparison between sFFR and SFR



- ✓ Highest SE→SOWF+IFR3;
- ✓ SOWF + IFR3 is **53%** higher than the traditional IFR1 +IFR3;
- ✓ Nearly **30%** higher than SWF + IFR3, the same is true in SFR with stronger intra-cell interference.

And even in the SFR, the SE of SOWF + IFR3 is approximate to the other two algorithms in sFFR scenario.

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Conclusion

- ❑ Proposed a SE-optimal SOWF power allocation algorithm, **which incorporates each user's channel SINR correlation** to determine the power allocation scheme;
- ❑ Apply BFS in indentifying the cell group and allocate the frequency channel more efficiently.
- ❑ Near-optimal FLWF approach was proposed to reduce the complexity of computing the optimal power allocation scheme.

Outlook

- ❑ Future work involves developing more intelligent power allocation schemes with lower computational complexity.
- ❑ Analyzing system SE with imperfect CSI and incorporating additional energy efficiency or outage probability constraints at the UE or BS are other interesting avenues for future work.

Reference I

- Abbas, Ziaul Haq et al. (2020). "Enabling soft frequency reuse and Stienen's cell partition in two-tier heterogeneous networks: Cell deployment and coverage analysis". In: *IEEE Transactions on Vehicular Technology* 70.1, pp. 613–626.
- Garcia-Morales, Jan, Guillem Femenias, and Felip Riera-Palou (2019). "Higher order sectorization in FFR-aided OFDMA cellular networks: Spectral-and energy-efficiency". In: *IEEE Access* 7, pp. 11127–11139.
- Han, Shuangfeng et al. (2017). "Big data enabled mobile network design for 5G and beyond". In: *IEEE Communications Magazine* 55.9, pp. 150–157.
- Novlan, Thomas David et al. (2011). "Analytical evaluation of fractional frequency reuse for OFDMA cellular networks". In: *IEEE Transactions on wireless communications* 10.12, pp. 4294–4305.
- Ren, Jie and Kai-Kit Wong (2018). "Cognitive radio made practical: Forward-lookingness and calculated competition". In: *IEEE Access* 7, pp. 2529–2548.

Reference II

Sun, Shu et al. (2018). "Analytical framework of hybrid beamforming in multi-cell millimeter-wave systems". In: *IEEE Transactions on Wireless Communications* 17.11, pp. 7528–7543.

Thank You
Q & A

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